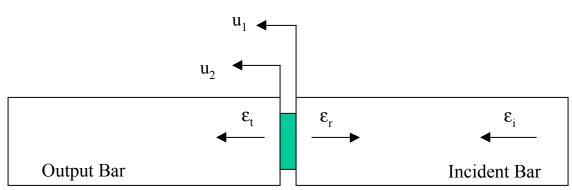
Theory of the Split Hopkinson Bar

The Split Hopkinson Bar (SHB) apparatus consists of a striker bar, an incident bar, the specimen under test and the output bar. A rectangular compression wave of well defined amplitude and length is generated in the incident bar when it is struck by the striker bar. When this wave reaches the specimen some of it is transmitted into the specimen and on into the output bar, and some is reflected back to the incident bar. The system is shown schematically in Figure 1. Using a one-dimensional wave propagation analysis [1], it is possible to determine high strain rate stress-strain curves from measurements of strain in the incident and output bars.



Waves travelling in the bars are, to a good approximation, governed by the one dimensional wave equation,

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

which has the solution

$$u = f(x - ct) + g(x + ct) = u_i + u_r \qquad (2)$$

where u_i is the incident wave and u_r is the reflected wave.

Assuming a one-dimensional system, the strain in the incident rod is, by definition,

$$\varepsilon = \frac{\partial u}{\partial x} \tag{3}$$

Therefore strain in the incident bar can be determined by differentiating equation (2):

$$\varepsilon = f' + g' = \varepsilon_i + \varepsilon_r \tag{4}$$

The velocity of any point on the bar can be determined by differentiating equation (2) with respect to time. From equation (4),

$$\dot{u} = c(-f' + g') = c(-\varepsilon_i + \varepsilon_r) \tag{5}$$

For the compressional SHB test the output bar only has a single wave, the transmitted wave, propagating in it (until reflection occurs at the end, which occurs after the time of interest). Therefore the velocity in the output bar is

$$\dot{u} = -c\varepsilon_t \tag{6}$$

The strain rate in the specimen is calculated from

$$\dot{\varepsilon} = \frac{(\dot{u}_1 - \dot{u}_2)}{l_s} \tag{7}$$

where l_s is the instantaneous length of the specimen. Combining equations (5), (6) and (7), the strain rate can be calculated from the strains by:

$$\dot{\varepsilon} = \frac{c}{l_s}(-\varepsilon_i + \varepsilon_r + \varepsilon_t) \tag{8}$$

The forces in the two bars are

$$F_1 = AE(\varepsilon_i + \varepsilon_r) \tag{9}$$
$$F_2 = AE\varepsilon_t \tag{10}$$

where A is the cross-sectional area of the bar and E is the Young’s modulus.

The analysis for the SHB assumes that after some initial ringing, the specimen is deforming uniformly and the forces are the same on each side of the specimen. Equating equations (9) and (10) yields

$$\varepsilon_t = \varepsilon_i + \varepsilon_r \tag{11}$$

Substituting (11) into (8) gives a relationship for calculating the strain rate in the specimen:

$$\dot{\varepsilon} = \frac{2c\varepsilon_r}{l_s} \tag{12}$$

The stress in the specimen is calculated by dividing the force in Equation (10) by the cross-sectional area of the specimen:

$$\sigma(t) = \frac{AE\varepsilon_t}{A_s} \tag{13}$$

Therefore the stress-strain curve can be determined from the two strain gage signals.

1. Gray, George T. (Rusty) III, High Strain-Rate Testing of Materials: The Split Hopkinson Bar. Methods in Materials Research, John Wiley Press, Oct. 1997.